Gyrokinetic formula and experimental examination of the electron-beam misalignment effect on the efficiency of a cylindrical-cavity gyrotron oscillator

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(Received 31 January 2000; published 23 January 2001)

By making use of the gyrokinetics of free-electron masers, the efficiency formula of a cylindrical-cavity gyrotron oscillator is presented, where the misalignment of the electron-beam axis to the cavity axis has been taken into account. Comparison with a recent experimental report [Int. J. Infrared and Millimeter Waves **19**, 1303 (1998)] is made, which confirms the creditability of the gyrokinetic theory.

DOI: 10.1103/PhysRevE.63.026501 PACS number(s): 41.60.Cr, 84.40.Ik, 52.59.Px

I. INTRODUCTION

As high-power, high-efficiency radiation sources in millimeter and submillimeter wave ranges, gyrotrons have significant applications, such as communications and electron cyclotron resonance heating on tokamaks and stellarators. The rf structure of gyrotrons most often used to date is a cylindrical cavity or a coaxial cavity. A cylindrical cavity has the advantage of large-volume power due to good heat diffusion, while a coaxial cavity has the peculiarity of suppressing multimode competition [1–4].

From the experimental point of view, both the cylindrical-cavity gyrotron and the coaxial-cavity gyrotron may have possible misalignments of the electron-beam axis to the cavity axis. The influence of this misalignment in a cylindrical-cavity gyrotron was studied in terms of analytical theory [5]. The gyrokinetics of the electron-beam misalignment effect in a coaxial-cavity gyrotron was derived [6]. Recently, the electron-beam misalignment effect was experimentally and numerically studied, and the experiment verified that even relatively small electron-beam misalignment will drastically deteriorate the performance of a cylindrical-cavity gyrotron, resulting in an efficiency reduction [7].

In this paper we present the formula of the electron-beam misalignment effect on the efficiency of a cylindrical-cavity gyrotron by means of the gyrokinetics of free-electron masers [8–14]. Then we compare the gyrokinetics with analytical theory [5], numerical simulation, and experiment [7].

II. GYROKINETIC FORMULA

We consider a relativistic electron beam drifting in a cylindrical cavity with an axial magnetostatic field $B_0\hat{e}_z$. Without loss of generality, we assume that (1) the axis of the electron beam has a misalignment to the cavity axis d, as shown in Fig. 1; and (2) the electron beam has no spreads and is tenuous enough so that its self-fields are negligible and the perturbation fields acting on the electron beam in the cavity can be treated like those of the rf fields in a cold cavity.

One of the theoretical approaches employed most often to investigate gyrodevices is the kinetic theory, which contains three main steps. First, based on Vlasov-Marxwell equations, the first-order perturbation distribution function of the relativistic electron beam f_1 is obtained by integrating the following equation along the unperturbed phase orbit [15]:

$$f_1 = -e\int_{t-z/v_\parallel}^{t}(\vec{E}_1' + \vec{v}\,' \times \vec{B}_1')\cdot\nabla_{\vec{p}} f_0\, dt', \tag{1}$$

where $\vec{v}$ and $\vec{p}$ are the velocity and momentum of the electron beam, e is the electron's negative charge, f_0 is the equilibrium distribution function of the electrons, $\vec{E}_1$ and $\vec{B}_1$ are the electric and magnetic fields of a TE_{mn} mode in the cavity which act on the electron beam as the perturbation fields, and the prime denotes the corresponding values defined on the unperturbed phase orbits of the electrons. Second, the perturbed current density $\vec{J}_1$ is performed by

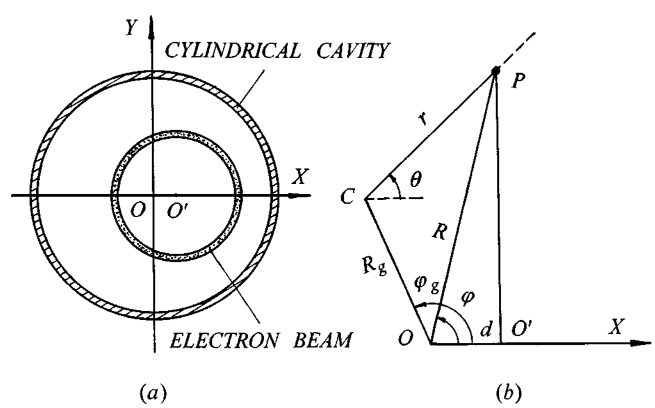

FIG. 1. (a) Cross-sectional view of the cylindrical-cavity gyrotron configuration with an electron-beam misalignment $\overline{O}\,\overline{O}' = d$. (b) The coordinate system employed in the present paper, where P is the location of the considered electron, and C is its guiding center.

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$$\vec{J}_1 = \int e n_0 \vec{v} f_1 d^3 p, \tag{2}$$

where n_0 is the equilibrium electron-beam density. Third, the energy transfer per unit time from the electron beam to the rf fields, P, is calculated from

$$P = -\frac{1}{2}\mathrm{Re}\left(\int_0^L dz \int_{Se} \vec{J}_1 \cdot \vec{E}_1^* ds \right), \tag{3}$$

where L is the cavity length, Se is the cross section of the electron beam, and $\vec{E}_1^*$ is the complex conjugate of the rf electric field $\vec{E}_1$. Inserting Eq. (3) into Eq. (2) yields

$$P = -\frac{1}{2}\mathrm{Re}\left(e n_0 \int_0^L dz \int_{Se} \int_p f_1 \vec{v} \cdot \vec{E}_1^* p_\perp \times R dR d\varphi dp_\perp dp_\parallel d\Phi \right), \tag{4}$$

where Φ is the momentum angle in momentum space.

In order to process the above integrals, the gyrokinetics of free-electron masers was developed [8–14]. Supposing that the electron's transverse position $\vec{R}$ is compounded by the radius vector from the electron to its guiding center $\vec{r}$ and the radius vector from the guiding center to the origin of the coordinates system $\vec{R}_g$ (i.e., $\vec{R} = \vec{r} + \vec{R}_g$), the gyrokinetic treatment chose the coordinates of the guiding center (R_g and φ_g) and the cyclotron angle θ as the gyrokinetic variables. Then three techniques were employed. The first technique is to rewrite the rf fields of the TE_{mn} mode ''felt'' by the electrons in terms of the cyclotron radius r, the cyclotron angle θ, and the coordinates of the guiding center R_g and φ_g. The second technique is that the equilibrium distribution function f_0 is expressed in terms of the transverse momentum $p_\perp$, and the coordinates of the guiding center R_g and φ_g, which were strictly shown to be the constants of motion in equilibrium [14]. In this way the integrand of the perturbation distribution function f_1 in Eq. (1) becomes an exponential function to t', and consequently, the integral can be performed easily. The third technique is to make the transformation

$$R dR d\varphi dz p_\perp dp_\perp dp_\parallel d\Phi = R_g p_\perp dR_g d\varphi_g dp_\perp dp_\parallel d\theta \tag{5}$$

or [10]

$$R dR d\varphi d\Phi = R_g dR_g d\varphi_g d\theta, \tag{6}$$

since the integrands, f_1 and $\vec{E}_1$ in Eq. (4), have been expressed by the cyclotron radius r, cyclotron angle θ, and the coordinates of the guiding center (R_g, φ_g). After straightforward algebraic manipulation, as done in Ref. [6], we obtain

$$P = -\frac{\pi n_0 v_{\parallel 0} R_0 r_0 e^2 A_m^2}{2\omega m_0 \gamma_0}(S_1 + S_2 + S_3 + S_4), \tag{7}$$

where

$$S_1 = \sum_{q=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} J_q^2(k_\perp d) J_l'(k_\perp r_0) J_l''(k_\perp r_0) J_{m-l+q}^2(k_\perp R_0) \times (A_{+l} + A_{-l}), \tag{8}$$

$$S_2 = \sum_{q=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \frac{v_{\perp 0}^2}{c^2} J_q^2(k_\perp d) J_l'^2(k_\perp r_0) J_{m-l+q}^2(k_\perp R_0) \times (B_{+l} + B_{-l}), \tag{9}$$

$$S_3 = \sum_{q=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} J_q^2(k_\perp d) J_l'^2(k_\perp r_0) J_{m-l+q}^2(k_\perp R_0) \times (C_{+l} + C_{-l}), \tag{10}$$

$$S_4 = \sum_{q=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} (D_{+l} + D_{-l}), \tag{11}$$

with

$$A_{\pm l} = 2 l \omega_{c0} F_{\pm l}, \tag{12}$$

$$B_{\pm l} = \frac{4(\omega \mp l\omega_{c0})(\omega^2 - k_\parallel^2 c^2)}{(\omega \mp l\omega_{c0} \pm k_\parallel v_{\parallel 0})(\omega \mp l\omega_{c0} \mp k_\parallel v_{\parallel 0})} F_{\pm l} + \frac{k_\parallel v_{\parallel 0} p \pi (-1)^p [\omega^2 - (\omega \mp l\omega_{c0})^2 c^2 / v_{\parallel 0}^2]}{(\omega \mp l\omega_{c0} \pm k_\parallel v_{\parallel 0})^2 (\omega \mp l\omega_{c0} \mp k_\parallel v_{\parallel 0})^2} \times \sin\left(\frac{\omega \mp l\omega_{c0}}{k_\parallel v_{\parallel 0}} p\pi \right), \tag{13}$$

$$C_{\pm l} = 2\left[\left(\frac{v_{\perp 0}}{v_{\parallel 0}} \right)^2 (\omega \mp l\omega_{c0}) \pm l\omega_{c0} \right] F_{\pm l}, \tag{14}$$

$$D_{\pm l} = \mp 2 k_\perp v_{\perp 0} J_q^2(k_\perp d) F_{\pm l} \Bigg\{ (m-l) \times (R_0/r_0)^{-1} J_l'^2(k_\perp r_0) J_{m-l+q}(k_\perp R_0) J_{m-l+q}'(k_\perp R_0) - l J_l(k_\perp r_0) J_l'(k_\perp r_0) \left[J_{m-l+q}'^2(k_\perp R_0) + \left[\frac{(m-l)^2}{(k_\perp R_0)^2} - 1 \right] J_{m-l+q}^2(k_\perp R_0) \right] \Bigg\}, \tag{15}$$

$$F_{\pm l} = \frac{k_\parallel^2 v_{\parallel 0}^2}{(\omega \mp l\omega_{c0} \pm k_\parallel v_{\parallel 0})^2 (\omega \mp l\omega_{c0} \mp k_\parallel v_{\parallel 0})^2} \times \left[(-1)^p \cos\left(\frac{\omega \mp l\omega_{c0}}{k_\parallel v_{\parallel 0}} p\pi \right) - 1 \right], \tag{16}$$

where m_0 is the electron's rest mass, γ_0, $v_{\perp 0}$, $v_{\parallel 0}$, R_0, and $2r_0$ are the e-beam's average relativistic energy factor, transverse velocity, longitudinal velocity, average radius and thickness, respectively, $\omega_{c0} = |e| B_0 / (\gamma_0 m_0)$ is the electrons' cyclotron frequency in the axial magnetostatic field B_0, c is the speed of light in vacuum, and ω, $k_\perp$, and $k_\parallel$ are the wave angular frequency, transverse wave number, and axial wave

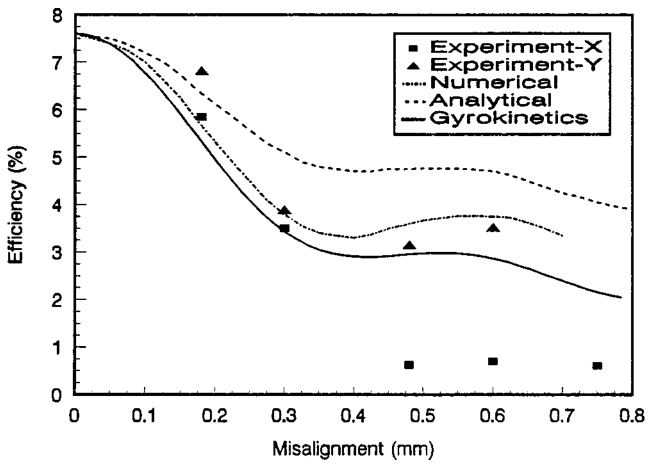

FIG. 2. The effect of the electron-beam misalignment on the efficiency of a $TE_{3,3,1}$-mode gyrotron oscillator at a frequency of 335 GHz, where the solid line denotes results from the gyrokinetic formula presented in this paper, the broken line is from the analytical theory in Ref. [5], the dash-dotted line is from the numerical simulation in Ref. [7], and the experimental measurements are duplicated from Fig. 5 in Ref. [7]. It was claimed in Ref. [7] that the poor efficiency for greater misalignments in the *Y* direction (marked by the squares) was attributed to imperfect coupling between the step-cut launcher and the transmission line to the detecting system. It is believed that if there is no imperfect coupling, the effect of the misalignment in the *Y* direction should be close to that in the *X* direction (marked by the triangles).

number of the $TE_{m,n,q}$ mode. J_m and J'_m are Bessel functions of the first kind with order *m* and its derivative with respect to the whole argument, respectively.

Supposing the power of the electron beam to be $P_{\text{beam}} = I_b V_b$, where I_b and V_b are the electron-beam current and voltage, we define the transfer efficiency

$$\eta = \frac{P}{P_{\text{beam}}}. \tag{17}$$

Then we obtain

$$\eta(d) = \eta(d=0)\,\frac{P(d)}{P(d=0)}, \tag{18}$$

where $d=0$ means no misalignment between the electron-beam axis and the cavity axis.

III. COMPARISONS

Recently, an experiment with a cylindrical-cavity gyrotron oscillator was reported by a Japanese research group [7]. In the experiment the device operated in a mode of $TE_{3,3,1}$ at a frequency of 335 GHz. The cylindrical cavity had a radius of 1.615 mm and a length of 14.5 mm. The electron beam had a voltage of 40 keV and an average radius of 0.81 mm, and the ratio of the transverse velocity to the parallel velocity to be 1.5. The operating magnetic field was 12.79 T. In their paper the authors presented comparison of their numerical simulations, as well as the analytical theory in Ref. [5], with the experimental observations.

Figure 2 shows the electron-beam misalignment effect on the efficiency, where the results come from the analytical theory [5] (broken line), the gyrokinetics presented in this paper (solid line), the numerical simulation (dash-dotted line), and the experimental measurements (denoted by triangle and square symbols) [7]. It should be pointed out that the curves of the analytical theory and numerical simulation, along with the experimental data, are duplicated from Fig. 5 in Ref. [7]. The result of gyrokinetics only considers the fundamental cyclotron harmonic $(l=-1)$. The squares and triangles correspond to the experimental measurements, where the electron-beam misalignment occurred in the *X* and *Y* directions, respectively. It can be seen from the figure that the gyrokinetics has a better agreement with the experiment than the analytical theory presented in Ref. [5], and closes up the numerical simulation before the misalignment to be 0.4 mm (i.e., $d/R_{\text{out}} < 25\%$). It was stated by the researchers in Ref. [7] that the poor efficiency for greater misalignments along the *Y* direction can be partly attributed to imperfect coupling between the step-cut launcher and the transmission line to the detecting system. In fact, it is believed that the effect of misalignment in the *Y* direction should be close to the one in the *X* direction for a cylindrical-cavity gyrotron oscillator, if there is no imperfect coupling [16]. Therefore, the figure shows that the gyrokinetics seems to still have an acceptable accuracy for greater misalignment $(47\% > d/R_{\text{out}} > 25\%)$.

IV. CONCLUSIONS

In this paper we have presented a gyrokinetic formula of the electron-beam misalignment effect on the efficiency of a cylindrical-cavity gyrotron oscillator. Comparison with the experiment has confirmed that the gyrokinetics is credible even if the electron-beam misalignment is great. Morefore, the theory in the present paper can be extended to a cylindrical-cavity amplifier, as well as to the situation of a TM mode.

ACKNOWLEDGMENTS

The author is grateful to Dr. I. Ogawa for useful discussion. This work was supported by the National Natural Science Foundation of China.

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